11 UNRESTRAINED BEAM DESIGN – I

1.0 INTRODUCTION

Generally, a beam resists transverse loads by bending action. In a typical building frame, main beams are employed to span between adjacent columns; secondary beams when used – transmit the floor loading on to the main beams. In general, it is necessary to consider only the bending effects in such cases, any torsional loading effects being relatively insignificant. The main forms of response to uni-axial bending of beams are listed in Table 1.

Under increasing transverse loads, beams of category 1 [Table1] would attain their full plastic moment capacity. This type of behaviour has been covered in an earlier chapter. Two important assumptions have been made therein to achieve this ideal beam behaviour. They are:

- ♦ The compression flange of the beam is restrained from moving laterally, and
- ♦ Any form of local buckling is prevented.

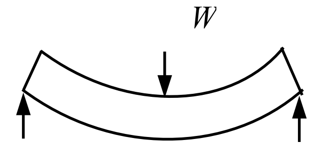
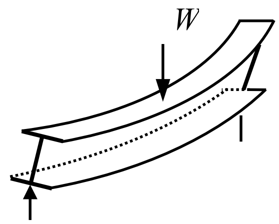
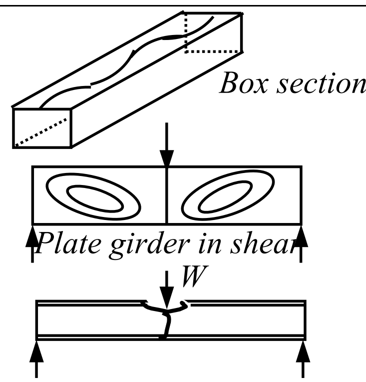
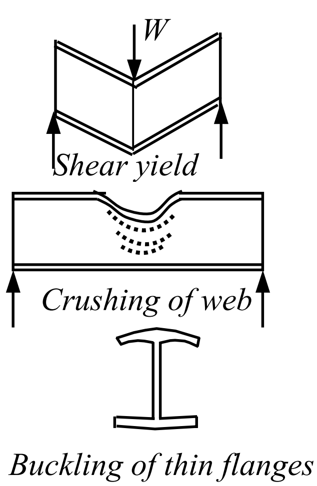
If the laterally unrestrained length of the compression flange of the beam is relatively long as in category 2 of Table 1, then a phenomenon, known as *lateral buckling* or *lateral torsional buckling* of the beam may take place. The beam would fail well before it could attain its full moment capacity. This phenomenon has a close similarity to the Euler buckling of columns, triggering collapse before attaining its squash load (full compressive yield load).

Lateral buckling of beams has to be accounted for at all stages of construction, to eliminate the possibility of premature collapse of the structure or component. For example, in the construction of steel-concrete composite buildings, steel beams are designed to attain their full moment capacity based on the assumption that the flooring would provide the necessary lateral restraint to the beams. However, during the erection stage of the structure, beams may not receive as much lateral support from the floors as they get after the concrete hardens. Hence, at this stage, they are prone to lateral buckling, which has to be consciously prevented.

Beams of category 3 and 4 given in Table 1 fail by local buckling, which should be prevented by adequate design measures, in order to achieve their capacities. The method of accounting for the effects of local buckling on bending strength was discussed in an earlier chapter.

In this chapter, the conceptual behaviour of laterally unrestrained beams is described in detail. Various factors that influence the lateral buckling behaviour of a beam are explained. The design procedure for laterally unrestrained beams is also included.

Table 1 Main failure modes of hot-rolled beams

Category	Mode		Comments
1	Excessive bending triggering collapse		This is the basic failure mode provided (1) the beam is prevented from buckling laterally,(2) the component elements are at least compact, so that they do not buckle locally. Such "stocky" beams will collapse by plastic hinge formation.
2	Lateral torsional buckling of long beams which are not suitably braced in the lateral direction.(i.e. "un restrained" beams)		Failure occurs by a combination of lateral deflection and twist. The proportions of the beam, support conditions and the way the load is applied are all factors, which affect failure by lateral torsional buckling.
3	Failure by local buckling of a flange in compression or web due to shear or web under compression due to concentrated loads		Unlikely for hot rolled sections, which are generally stocky. Fabricated box sections may require flange stiffening to prevent premature collapse. Web stiffening may be required for plate girders to prevent shear buckling. Load bearing stiffeners are sometimes needed under point loads to resist web buckling.
4	Local failure by (1) shear yield of web (2) local crushing of web (3) buckling of thin flanges.		Shear yield can only occur in very short spans and suitable web stiffeners will have to be designed. Local crushing is possible when concentrated loads act on unstiffened thin webs. Suitable stiffeners can be designed. This is a problem only when very wide flanges are employed. Welding of additional flange plates will reduce the plate b / t ratio and thus flange buckling failure can be avoided.

2.0 SIMILARITY OF COLUMN BUCKLING AND LATERAL BUCKLING OF BEAMS

It is well known that slender members under compression are prone to instability. When slender structural elements are loaded in their strong planes, they have a tendency to fail by buckling in their weaker planes. Both axially loaded columns and transversely loaded beams exhibit closely similar failure characteristics due to buckling.

Column buckling has been dealt with in detail in an earlier chapter. In this section, lateral buckling of beams is described and its close similarity to column buckling is brought out.

Consider a simply supported and laterally unsupported (except at ends) beam of "short-span" subjected to incremental transverse load at its mid section as shown in Fig.1 (*a*). The beam will deflect downwards i.e. in the direction of the load [Fig. 1(*b*)].

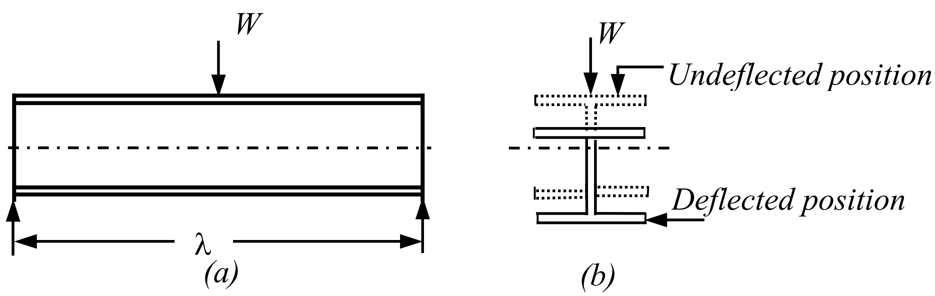


Fig. 1(a) Short span beam, (b) Vertical deflection of the beam.

The direction of the load and the direction of movement of the beam are the same. This is similar to a short column under axial compression. On the other hand, a "long-span" beam [Fig.2 (*a*)], when incrementally loaded will first deflect downwards, and when the load exceeds a particular value, it will tilt sideways due to instability of the compression flange and rotate about the longitudinal axis [Fig. 2(*b*)].

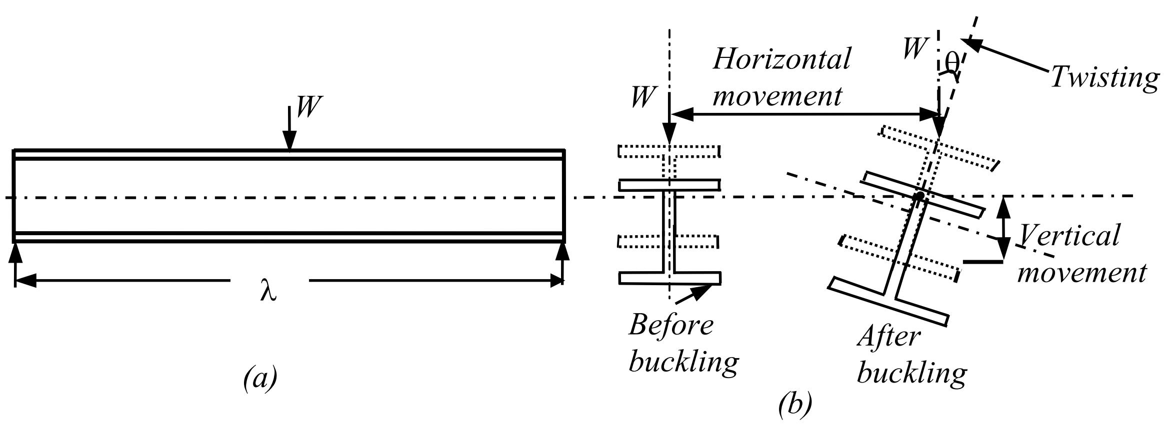


Fig. 2(a) Long span beam, (b) Laterally deflected shape of the beam

The three positions of the beam cross-section shown in Fig. 2(*b*) illustrate the displacement and rotation that take place as the midsection of the beam undergoes lateral torsional buckling. The characteristic feature of lateral buckling is that the entire cross section rotates as a rigid disc without any cross sectional distortion. This behaviour is very similar to an axially compressed long column, which after initial shortening in the axial direction, deflects laterally when it buckles. The similarity between column buckling and beam buckling is shown in Fig. 3.

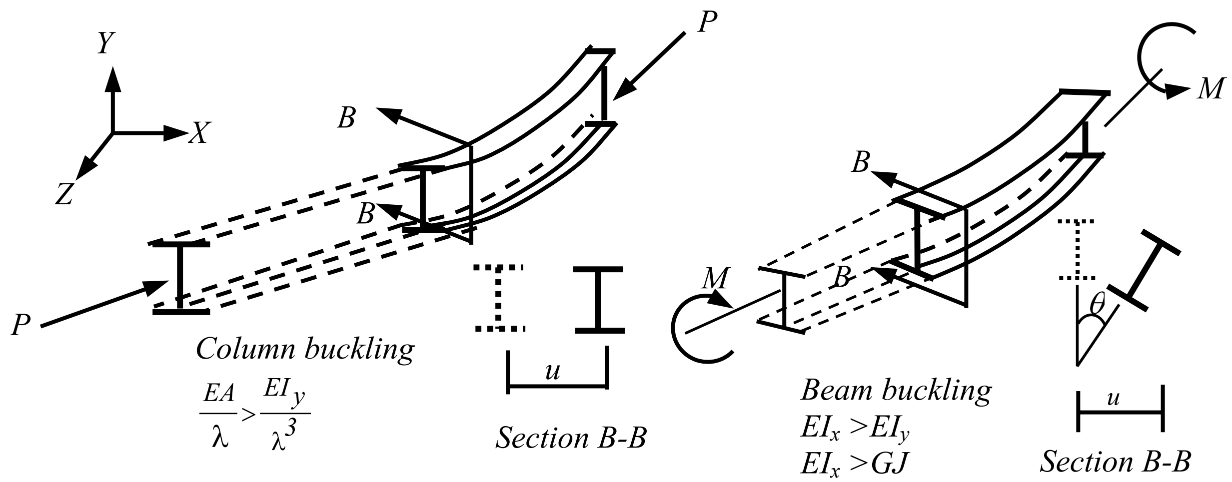


Fig. 3 Similarity of column buckling and beam buckling

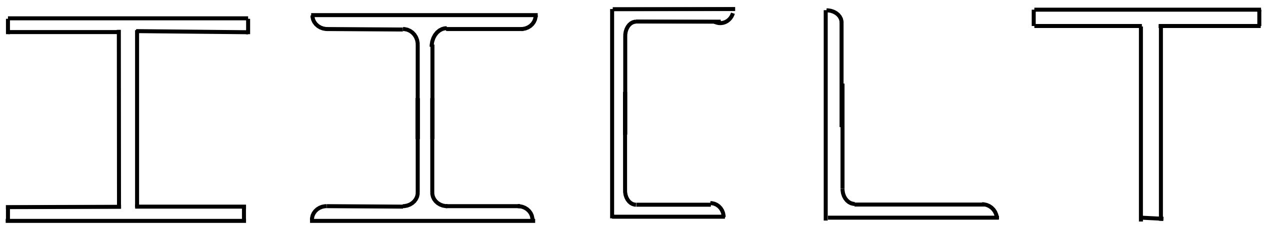
In the case of axially loaded columns, the deflection takes place sideways and the column buckles in a pure flexural mode. A beam, under transverse loads, has a part of its cross section in compression and the other in tension. The part under compression becomes unstable while the tensile stresses elsewhere tend to stabilize the beam and keep it straight. Thus, beams when loaded exactly in the plane of the web, at a particular load, will fail suddenly by deflecting sideways and then twisting about its longitudinal axis [Fig.3]. This form of instability is more complex (compared to column instability) since the lateral buckling problem is 3-dimensional in nature. It involves coupled lateral deflection and twist i.e., when the beam deflects laterally, the applied moment exerts a torque about the deflected longitudinal axis, which causes the beam to twist. The bending moment at which a beam fails by lateral buckling when subjected to a uniform end moment is called its *elastic critical moment* (M_{cr}). In the case of lateral buckling of beams, the elastic buckling load provides a close upper limit to the load carrying capacity of the beam. It is clear that lateral instability is possible only if the following two conditions are satisfied.

- The section possesses different stiffness in the two principal planes, and
- The applied loading induces bending in the stiffer plane (about the major axis).

Similar to the columns, the lateral buckling of unrestrained beams, is also a function of its slenderness.

3.0 INFLUENCE OF CROSS SECTIONAL SHAPE ON LATERAL TORSIONAL BUCKLING

Structural sections are generally made up of either open or closed sections. Examples of open and closed sections are shown in Fig. 4.



Open sections

Closed sections

Fig. 4 Open and closed sections

Cross sections, employed for columns and beams (I and channel), are usually open sections in which material is distributed in the flanges, i.e. away from their centroids, to improve their resistance to in-plane bending stresses. Open sections are also convenient to connect beams to adjacent members. In the ideal case, where the beams are restrained laterally, their bending strength about the major axis forms the principal design consideration. Though they possess high major axis bending strength, they are relatively weak in their minor axis bending and twisting.

The use of open sections implies the acceptance of low torsional resistance inherent in them. No doubt, the high bending stiffness (EI_x) available in the vertical plane would result in low deflection under vertical loads. However, if the beam is loaded laterally, the deflections (which are governed by the lower EI_y rather than the higher EI_x) will be very much higher. From a conceptual point of view, the beam has to be regarded as an element having an enhanced tendency to fall over on its weak axis.

In contrast, closed sections such as tubes, boxes and solid shafts have high torsional stiffness, often as high as 100 times that of an open section. The hollow circular tube is the most efficient shape for torsional resistance, but is rarely employed as a beam element on account of the difficulties encountered in connecting it to the other members and lesser efficiency as a flexural member. The influence of sectional shapes on the lateral strength of a beam is further illustrated in a later Section.

4.0 LATERAL TORSIONAL BUCKLING OF SYMMETRIC SECTIONS

As explained earlier, when a beam fails by lateral torsional buckling, it buckles about its weak axis, even though it is loaded in the strong plane. The beam bends about its strong axis up to the critical load at which it buckles laterally [Fig. 5(*a*) and 5(*b*)].

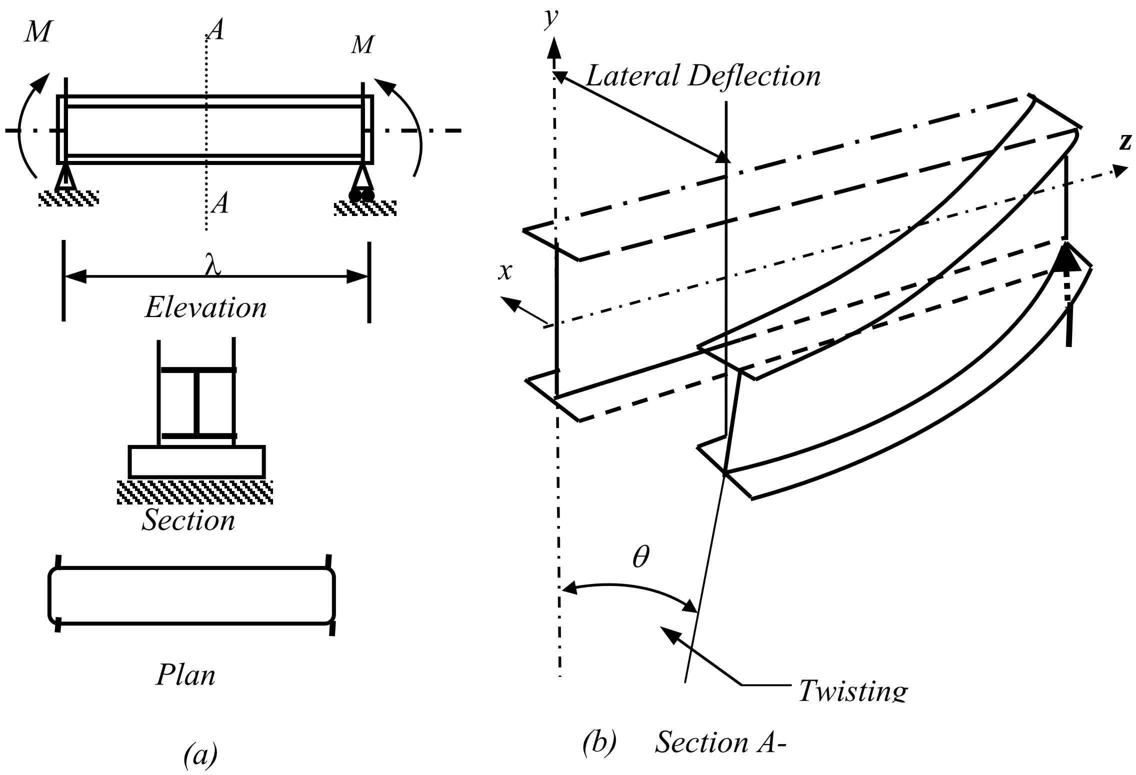


Fig. 5(a) Original beam (b) laterally buckled beam

For the purpose of this discussion, the lateral torsional buckling of an I-section is considered with the following assumptions.

1. The beam is initially undistorted
2. Its behaviour is elastic (no yielding)
3. It is loaded by equal and opposite end moments in the plane of the web.
4. The loads act in the plane of the web only (there are no externally applied lateral or torsional loads)
5. The beam does not have residual stresses
6. Its ends are simply supported vertically and laterally.

Obviously, in practice, the above ideal conditions are seldom met. For example, rolled sections invariably contain residual stresses. The effects of the deviations from the ideal case are discussed in a later Section.

The critical bending moment capacity attained by a symmetric I beam subjected to equal end moments undergoing lateral torsional buckling between points of lateral or torsional support is a function of two torsional characteristics of the specific cross-section: the pure torsional resistance under uniform torsion and the warping torsional resistance

M_{cr} = [(torsional resistance)^2 + (warping resistance)^2]$^{1/2}$

$$M_{cr} = \frac{\pi}{\lambda}\left[E I_y G J + \frac{\pi^2 E I_y \Gamma}{\lambda^2}\right]^{\frac{1}{2}} \quad 1(a)$$

This may be rewritten as

$$M_{cr} = \frac{\pi}{\lambda}\left(E I_y G J\right)^{\frac{1}{2}}\left[1 + \frac{\pi^2 E \Gamma}{\lambda^2 G J}\right]^{\frac{1}{2}} \quad 1(b)$$

where, EI_y is the minor axis flexural rigidity
GJ is the torsional rigidity
$E\Gamma$ is the warping rigidity

The torsion that accompanies lateral buckling is always non-uniform. The critical bending moment, M_{cr} is given by Eqn.1 (a).

It is evident from Eqn.1 (*a*) that the flexural and torsional stiffness of the member relate to the lateral and torsional components of the buckling deformations. The magnitude of the second square root term in Eqn.1 (*b*) is a measure of the contribution of warping to the resistance of the beam. In practice, this value is large for short deep girders. For long shallow girders with low warping stiffness, $\Gamma \approx 0$ and Eqn. 1(*b*) reduces to

$$M_{cr} = \frac{\pi}{\lambda}\left(E I_y G J\right)^{\frac{1}{2}} \quad (2)$$

An I-section composed of very thin plates will posses very low torsional rigidity (since J depends on third power of thickness) and both terms under the root will be of comparable magnitude. The second term is negligible compared to the first for the majority of hot rolled sections. But light gauge sections derive most of the resistance to torsional deformation from the warping action. The beam length also has considerable influence upon the relative magnitudes of the two terms as shown in the term $\pi^2 E\Gamma / \lambda^2 GJ$. Shorter and deep beams ($\pi^2 E\Gamma / \lambda^2 GJ$ term will be large) demonstrate more warping resistance, whereas, the term will be small for long and shallow beams. Eqn. (1) may be rewritten in a simpler form as given below.

$$M_{cr} = \alpha \left(E I_y G J\right)^{\frac{1}{2}}\left[\frac{\pi}{\lambda}\left(1 + \frac{\pi^2}{B^2}\right)^{\frac{1}{2}}\right] \quad (3)$$

where $B^2 = \lambda^2 G J / E \Gamma$ 3(a)

$$M_{cr} = \alpha(E\, I_y\, G\, J)^{1/2}\, \gamma \qquad (4)$$

where $\gamma = \pi/\lambda\ (1+\pi^2 / B^2)^{1/2}$ 4(a)

Eqn. (4) is a product of three terms: the first term, α, varies with the loading and support conditions; the second term varies with the material properties and the shape of the beam; and the third term, γ, varies with the length of the beam. Eqn. (4) is regarded as the basic equation for lateral torsional buckling of beams. The influence of the three terms mentioned above is discussed in the following Section.

4.1 LATERAL TORSIONAL BUCKLING AS STIPULATED IN NEW IS: 800:

New IS: 800 take into account the effect of elastic critical moment for consideration of lateral torsional buckling and the stipulations are as follows:

a) Simplified equation for prismatic members made of standard rolled I-sections and welded doubly symmetric I-sections:

$$M_{cr} = \frac{\pi^2 E I_y h_f}{2 L_{LT}^2} \left[1 + \frac{1}{20} \left(\frac{L_{LT} / r_y}{h_f / t_f} \right)^2 \right]^{0.5} = \beta_b Z_p f_{cr,b}$$

I_t = torsional constant = $\sum b_i t_i^3 / 3$ for open section

I_w = warping constant

I_y, r_y = moment of inertia, radius of gyration about the weak axis, respectively

L_{LT} = effective length for lateral torsional buckling (**8.3**)

h_f = Center to center distance between flanges

t_f = thickness of the flange

b) For doubly symmetric prismatic beams

The elastic critical moment corresponding to lateral torsional buckling of a doubly symmetric prismatic beam subjected to uniform moment in the unsupported length and torsionally restraining lateral supports is given by

$$M_{cr} = \frac{\pi^2 E I_y}{(KL)^2} \left[\frac{I_w}{I_y} + \frac{G\, I_t\, (KL)^2}{\pi^2 E\, I_y} \right]^{0.5}$$

Where

I_y, I_w, I_t = Moment of inertia about the minor axis, warping constant and St. Venants torsion constant of the cross section, respectively

G = Modulus of rigidity

KL = Effective length against lateral torsional buckling

c) *For sections symmetric about minor axis:*

In case of a beam which is symmetrical only about the minor axis, and bending about major axis, the elastic critical moment for lateral torsional buckling is given by the general equation,

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + \left(c_2\, y_g - c_3\, y_j \right)^2 \right]^{0.5} - \left(c_2\, y_g - c_3\, y_j \right) \right\}$$

[c_1, c_2, c_3 = factors depending upon the loading and end restraint conditions (Table F.1)
K, K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports].

The effective length factors K varies from 0.5 for complete restraint against rotation about weak axis to 1.0 for free rotate about weak axis, with 0.7 for the case of one end fixed and other end free. It is analogous to the effective length factors for compression members with end rotational restraint.

The K_w factor refers to the warping restraint. Unless special provisions to restrain warping of the section at the end lateral supports are made, K_w should be taken as 1.0.

y_g is the y-distance between the point of application of the load and the shear centre of the cross section and is positive when the load is acting towards the shear centre from the point of application

$y_j = y_s - 0.5 \int_A (z^2 - y^2)\, y\, dA / I_z$

y_s is the coordinate of the shear centre with respect to centroid, positive when the shear centre is on the compression side of the centroid

y, z are coordinates of the elemental area with respect to centroid of the section

The z_j can be calculated by using the following approximation

a) Plain flanges

$y_j = 0.8\,(2\beta_f - 1)\, h_y / 2.0$ (when $\beta_f > 0.5$)
$y_j = 1.0\,(2\beta_f - 1)\, h_y / 2.0$ (when $\beta_f \leq 0.5$)

b) Lipped flanges

$y_j = 0.8\,(2\beta_f - 1)\,(1 + h_L/h)\, h_y/2$ (when $\beta_f > 0.5$)
$y_j = (2\beta_f - 1)\,(1 + h_L/h)\, h_y/2$ (when $\beta_f \leq 0.5$)

Where,

h_L = height of the lip
h = overall height of the section
h_y = distance between shear centre of the two flanges of the cross section

The torsion constant I_t is given by

$I_t = \sum b_i t_i^3 / 3$ for open section

$= 4 A_e^2 / \sum (b/t)$ for hollow section

Where,

A_e = area enclosed by the section

b, t = breadth and thickness of the elements of the section respectively

The warping constant, I_w, is given by

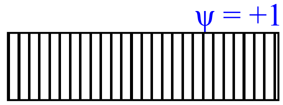
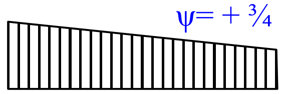
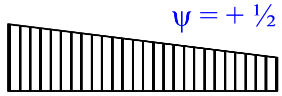
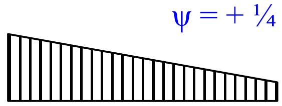
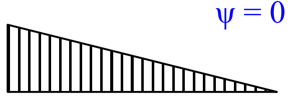
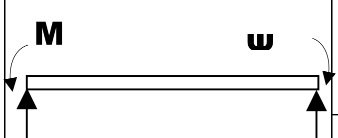
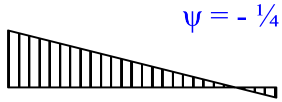
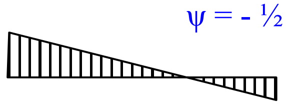
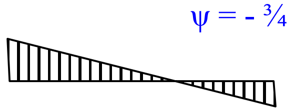
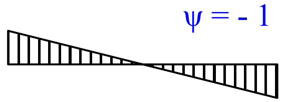
$I_w = (1-\beta_f)\,\beta_f\, I_y\, h_y^2$ for I sections mono-symmetric about weak axis

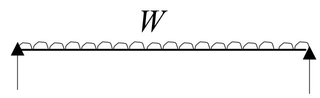
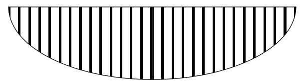
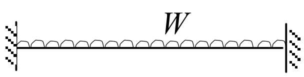
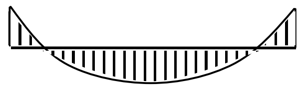
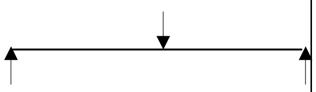
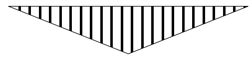
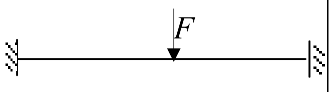
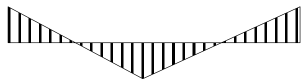
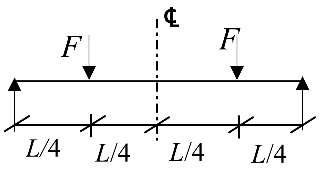
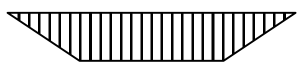
$= 0$ for angle, Tee, narrow rectangle section and approximately for hollow sections

$\beta_f = I_{fc} / (I_{fc} + I_{ft})$ where I_{fc}, I_{ft} are the moment of inertia of the compression and tension flanges, respectively, about the minor axis of the entire section

TABLE F.1 CONSTANTS c_1, c_2, AND c_3

(*Section* F.1.2)

Loading and Support Conditions	Bending Moment Diagram	Value of K	Constants c_1	Constants c_2	Constants c_3
	ψ = +1 	1.0 0.7 0.5	1.000 1.000 1.000	---	1.000 1.113 1.144
	ψ= + ¾ 	1.0 0.7 0.5	1.141 1.270 1.305	---	0.998 1.565 2.283
	ψ = + ½ 	1.0 0.7 0.5	1.323 1.473 1.514	---	0.992 1.556 2.271
	ψ = + ¼ 	1.0 0.7 0.5	1.563 1.739 1.788	---	0.977 1.531 2.235
	ψ = 0 	1.0 0.7 0.5	1.879 2.092 2.150	---	0.939 1.473 2.150
	ψ = - ¼ 	1.0 0.7 0.5	2.281 2.538 2.609	---	0.855 1.340 1.957
	ψ = - ½ 	1.0 0.7 0.5	2.704 3.009 3.093	---	0.676 1.059 1.546
	ψ = - ¾ 	1.0 0.7 0.5	2.927 3.009 3.093	---	0.366 0.575 0.837
	ψ = - 1 	1.0 0.7 0.5	2.752 3.063 3.149	---	0.000 0.000 0.000

Loading and Support Conditions	Bending Moment Diagram	Value of K	Constants		
			c_1	c_2	c_3
		1.0 0.5	1.132 0.972	0.459 0.304	0.525 0.980
		1.0 0.5	1.285 0.712	1.562 0.652	0.753 1.070
		1.0 0.5	1.365 1.070	0.553 0.432	1.780 3.050
		1.0 0.5	1.565 0.938	1.257 0.715	2.640 4.800
		1.0 0.5	1.046 1.010	0.430 0.410	1.120 1.390

5.0 FACTORS AFFECTING LATERAL STABILITY

The elastic critical moment, M_{cr}, as obtained in the previous Section, is applicable only to a beam of I section which is simply supported and subjected to end moments. This case is considered as the basic case for future discussion. In practical situations, support conditions, beam cross section, loading etc. vary from the basic case. The following sections elaborate on these variations and make the necessary modifications to the basic case for design purposes.

5.1 Support conditions

The lateral restraint provided by the simply supported conditions assumed in the basic case is the lowest and therefore M_{cr} is also the lowest. It is possible, by other restraint conditions, to obtain higher values of M_{cr}, for the same structural section, which would result in better utilization of the section and thus saving in weight of material. As lateral buckling involves three kinds of deformations, namely *lateral bending, twisting* and *warping*, it is feasible to think of various types of end conditions. But, the supports should either completely prevent or offer no resistance to each type of deformation. Solutions for partial restraint conditions are complicated. The effect of various support conditions is taken into account by way of a parameter called *effective length,* which is explained, in the next Section.

5.2 Effective length

The concept of effective length incorporates the various types of support conditions. For the beam with simply supported end conditions and no intermediate lateral restraint, the effective length is equal to the actual length between the supports. When a greater amount of lateral and torsional restraints is provided at supports, the effective length is less than the actual length and alternatively, the length becomes more when there is less restraint. The effective length factor would indirectly account for the increased lateral and torsional rigidities provided by the restraints.

For simply supported beams and girders of span length, L, where no lateral restraint to the compression flanges is provided, but where each end of the beam is restrained against torsion, the effective length L_{LT} of the lateral buckling shall be taken as in Table 2 (Table 8.3 of New IS: 800).

***Table 2 Effective length* for Simply Supported Beams**
(Table 8.3 of New IS: 800, Effective Length of Simply Supported Beams, L_{LT})

Conditions of restraint at supports		Loading condition	
Torsional restraint[1]	Warping Restraint[2]	Normal	Destabilising
Fully restrained	Both flanges fully restrained	0.70 L	0.85 L
Fully restrained	Compression flange fully restrained	0.75 L	0.90 L
Fully restrained	Both flanges fully restrained	0.80 L	0.95 L
Fully restrained	Compression flange partially restrained	0.85 L	1.00 L
Fully restrained	Warping not restrained in both flanges	1.00 L	1.20 L
Partially restrained by bottom flange support connection	Warping not restrained in both flanges	1.0 L+ 2D	1.2 L+2D
Partially restrained by bottom flange bearing support	Warping not restrained in both flanges	1.2 L+2D	1.4 L+2D
[1] Torsional restraint prevents rotation about the longitudinal axis [2] Warping restraint prevents rotation of the flange in its plane [3] D is the overall depth of the beam			

In simply supported beams with intermediate lateral restraints against lateral torsional buckling, the effective length for lateral torsional buckling L_{LT}, shall be taken as the length of the relevant segment in between the lateral restraints. The effective length shall be equal to 1.2 times the length of the relevant segment in between the lateral restraints.

Restraint against torsional rotation at supports in these beams may be provided by:

i) web or flange cleats, or
ii) bearing stiffeners acting in conjunction with the bearing of the beam, or
iii) lateral end frames or external supports provide lateral restraint to the compression flanges at the ends, or
iv) their being built into walls

5.3 Level of application of transverse loads

The lateral stability of a transversely loaded beam is dependent on the arrangement of the loads as well as the level of application of the loads with respect to the centroid of the cross section. Fig. 6 shows a centrally loaded beam experiencing either destabilising or restoring effect when the cross section is twisted.

A load applied above the centroid of the cross section causes an additional overturning moment and becomes more critical than the case when the load is applied at the centroid. On the other hand, if the load is applied below the centroid, it produces a stabilising effect. Thus, a load applied below or above the centroid can change the buckling load by *± 40%*. The location of the load application has no effect if a restraint is provided at the load point. For example, New IS: 800 takes into account the destabilising effect of top flange loading by using a notional effective length of 1.2 times the actual span to be used in the calculation of effective length (see Table 2).

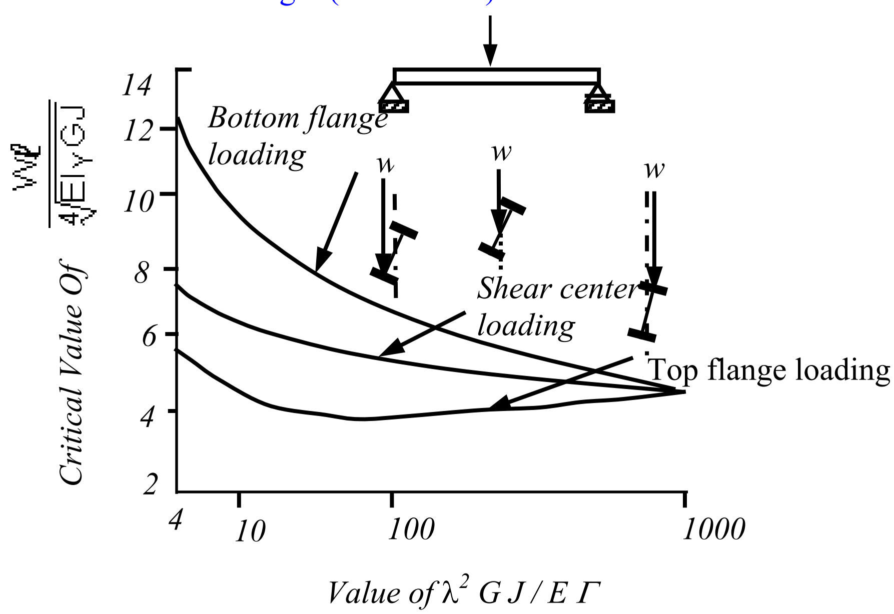


Fig.6 Effect of level of loading on beam stability

Provision of intermediate lateral supports can conveniently increase the lateral stability of a beam. With a central support, which is capable of preventing lateral deflection and twisting, the beam span is halved and each span behaves independently. As a result, the rigidity of the beam is considerably increased. This aspect is dealt in more detail in a later chapter.

5.4 Influence of type of loading

So far, only the basic case of beams loaded with equal and opposite end moments has been considered. But, in reality, loading patterns would vary widely from the basic case. The two reasons for studying the basic case in detail are: (1) it is analytically amenable, and (2) the loading condition is regarded as the most severe. Cases of moment gradient, where the end moments are unequal, are less prone to instability and this beneficial effect is taken into account by the use of ***"equivalent uniform moments".*** In this case, the basic design procedure is modified by comparing the elastic critical moment for the actual case with the elastic critical moment for the basic case. This process is similar to the effective length concept in strut problems for taking into account end fixity.

5.4.1 Loading applied at points of lateral restraint

While considering other loading cases, the variation of the bending moment within a segment (i.e. the length between two restraints) is assumed to be linear from M_{max} at one end to M_{min} at the other end as shown in Fig. 7.

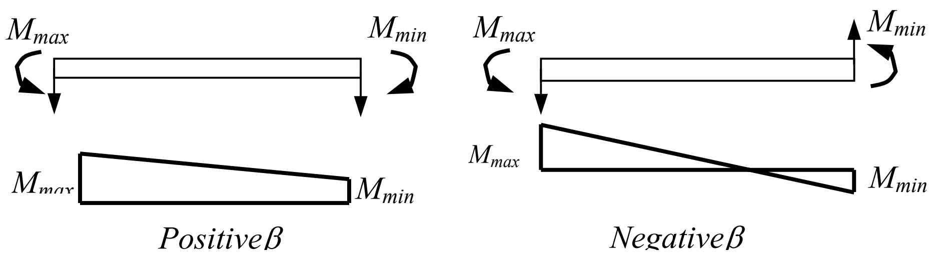


Fig. 7 Non uniform distribution of bending moment

The value of β is defined as $\beta = M_{min} / M_{max}$ $\quad (1.0 \geq \beta \geq -1.0)$ (5)

The value of β is positive for opposing moments at the ends (single curvature bending) and negative for moments of the same kind (double curvature bending). For a particular case of β, the value of M at which elastic instability occurs can be expressed as a ratio *'m'* involving the value of M_{cr} for the segment i.e. the elastic critical moment for $\beta = 1.0$. The ratio may be expressed as a single curve in the form:

$$m = 0.57 + 0.33\beta + 0.1\beta^2 \nless 0.43 \quad (6)$$

The quantity '*m'* is usually referred to as the *equivalent uniform moment factor.*

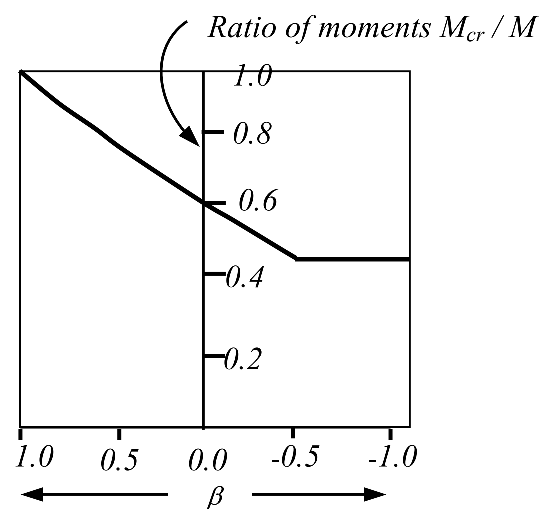


Fig. 8 'm' factor for equivalent uniform moment

The relationship is also expressed in Fig. 8. As seen from the figure, $m = 1.0$ for uniform moment and $m < 1.0$ for non uniform moment; therefore, beam with variation of moment over the unsupported length is less vulnerable to lateral stability as compared to that subjected to uniform moment. Its value is a measure of the intensity of the actual pattern of moments as compared with the basic case. In many cases, its value is dependent only on the shape of the moment diagram and a few examples are presented in Fig.9.

A good estimate of the critical moment due to the actual loading may be found using the proper value of m in the equation

$$M = (1 / m)\, M_{cr} \tag{7}$$

This approximation helps in predicting the buckling of the segments of a beam, which is loaded through transverse members preventing local lateral deflection and twist. Each segment is treated as a beam with unequal end moments and its elastic critical moments may be determined from the relationship given in Eqn.7. The critical moment of each segment can be determined and the lowest of them would give a conservative approximation to the actual critical moment.

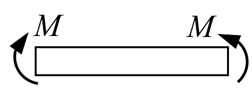
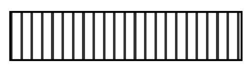
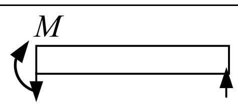
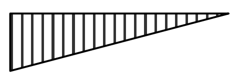
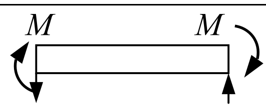
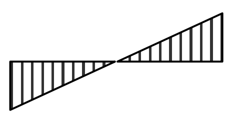
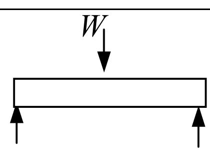
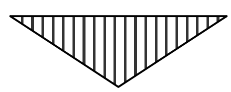
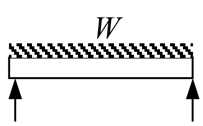
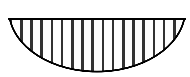
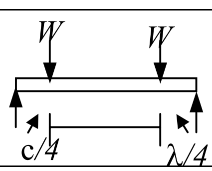
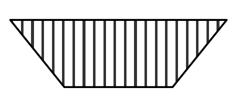
Beam and loads	Actual bending moment	M_{max}	m	Equivalent uniform moment
		M	1.0	
		M	0.57	
		M	0.43	
		$W\lambda/4$	0.74	
		$W\lambda^2/8$	0.88	
		$W\lambda/4$	0.96	

Fig. 9 Equivalent uniform moment

It may be noted here that the values of *'m'* apply only when the point of maximum moment occurs at one end of the segments of the beams with uniform cross section and equal flanges. In all other cases $m=1.0$. For intermediate values of β, m can be determined by Eqn. 6 or can be interpolated from Fig 8. The local strength at the more heavily stressed end also may be checked against plastic moment capacity, M_p as in Eqn. 8.

$$M_{max} \leq M_p. \qquad (8)$$

5.4.2 Use of *m* factors in design

As discussed earlier, the shape of the moment diagram influences the lateral stability of a beam. A beam design using uniform moment loading will be unnecessarily conservative. In order to account for the non-uniformity of moments, a modification of the moment may be made based on a comparison of the elastic critical moment for the basic case. This can be done in two ways. They are:

(i) Use equivalent uniform moment value $\overline{M} = m\, M_{max}$ (M_{max} is the larger of the two end moments) for checking against the buckling resistance moment M_b.

(ii) M_b value is determined using an effective slenderness ratio $\lambda'_{LT} = \lambda_{LT}\sqrt{m}$. (where λ_{LT} is the lateral torsional slenderness ratio and λ'_{LT} is the effective lateral torsional slenderness ratio).

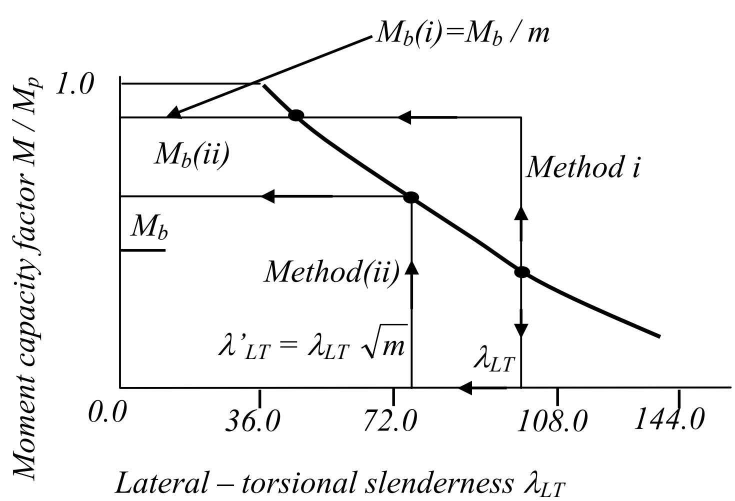


Fig. 10 Moment capacity of beams

The idea of lateral torsional slenderness λ_{LT} is introduced here to write the design capacity M_b as

$$\frac{M_b}{M_p} = f\left(\frac{1}{\lambda_{LT}^2}\right) \qquad (9)$$

where M_p is the fully plastic moment

The quantity λ_{LT} is defined by

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{p_y}}\sqrt{\frac{M_p}{M_{cr}}} \tag{10}$$

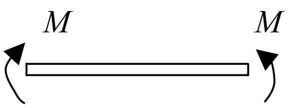
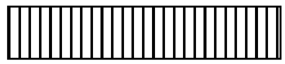
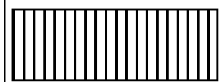
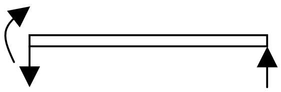
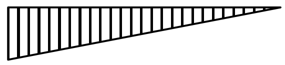
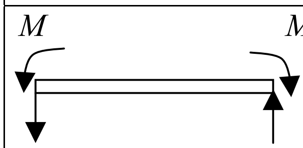
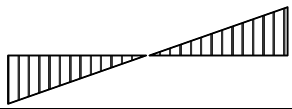
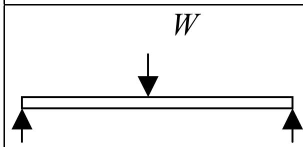
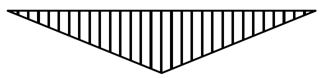
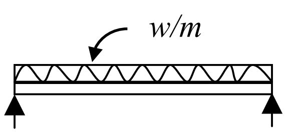
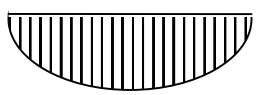
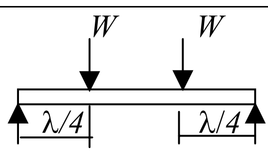
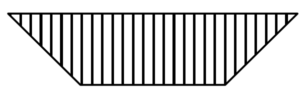
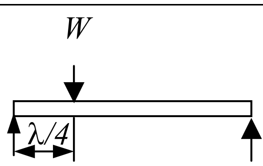
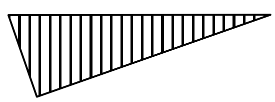
For a particular material (i.e particular E and p_y) the above equation can be considered as a product of c constant and $\sqrt{\frac{M_p}{M_{cr}}}\left(\overline{\lambda}_{LT}\right)$. The quantity $\overline{\lambda}_{LT}$ is called as the new defined slenderness ratio.

Buckling resistance moment, M_b is always less than the elastic critical moment, M_{cr}. Therefore, the second method is more conservative especially for low values of λ_{LT}. The two methods are compared in Fig. 10, where for the first case M_{max} is to be checked against $M_b\ / m$ and for the second case against M_b only. Method (i) is more suitable for cases where loads are applied only at points of effective lateral restraint. Here, the yielding is restricted to the supports; consequently, results in a small reduction in the lateral buckling strength. In order to avoid overstressing at one end, an additional check, $M_{max} < M_p$ should also be satisfied. In certain situations, maximum moment occurs within the span of the beam. The reduction in stiffness due to yielding would result in a smaller lateral buckling strength. In this case, the prediction according to method (i) based on the pattern of moments would not be conservative; here the method (ii) is more appropriate. In the second method, a correction factor n is applied to the slenderness ratio λ_{LT} and design strength is obtained for $n\lambda_{LT}$. It is clear from the above that $n = \sqrt{m}$. The slenderness correction factor is explained in the next section.

5.4.3 Slenderness correction factor

For situations, where the maximum moment occurs away from a braced point, e.g. when the beam is uniformly loaded in the span, a modification to the slenderness, λ_{LT}, may be used. The allowable critical stress is determined for an effective slenderness, $n\lambda_{LT}$, where n is the slenderness correction factor, as illustrated in Fig. 11 for a few cases of loading.

For design purposes, one of the above methods – either the moment correction factor method (m method) or slenderness correction factor method (n method) may be used. If suitable values are chosen for m and n, both methods yield identical results. The difference arises only in the way in which the correction is made; in the n factor method the slenderness is reduced to take advantage of the effect of the non- uniform moment, whereas, in the m factor method, the moment to be checked against lateral moment capacity, M_b, is reduced from M_{max} to $\overline{M}$ by the factor m. It is always safe to use $m = n$ =1 basing the design on uniform moment case. In any situation, either m = 1 or n= 1, i.e. any one method should be used.

Slenderness correction factor, *n*			
Load pattern	Actual bending moment	*n*	Equivalent uniform moment
		1.0	
		0.77	
		0.65	
		0.86	
		0.94	
		0.94	
		0.94	

5.5 Effect of cross-sectional shape

The shape of the cross-section of a beam is a very important parameter while evaluating its lateral buckling capacity. In other words, lateral instability can be reduced or even avoided by choosing appropriate sections. The effect of cross-sectional shape on lateral instability is illustrated in Fig. 12 for different type of section with same cross sectional area.

The figure shows that the I-section with the larger in-plane bending stiffness does not have matching stability. Box sections with high torsional stiffness are most suitable for beams. However, I-sections are commonly used due to their easy availability and ease of connections. Box sections are used as crane girders where the beam must be used in a laterally unsupported state.

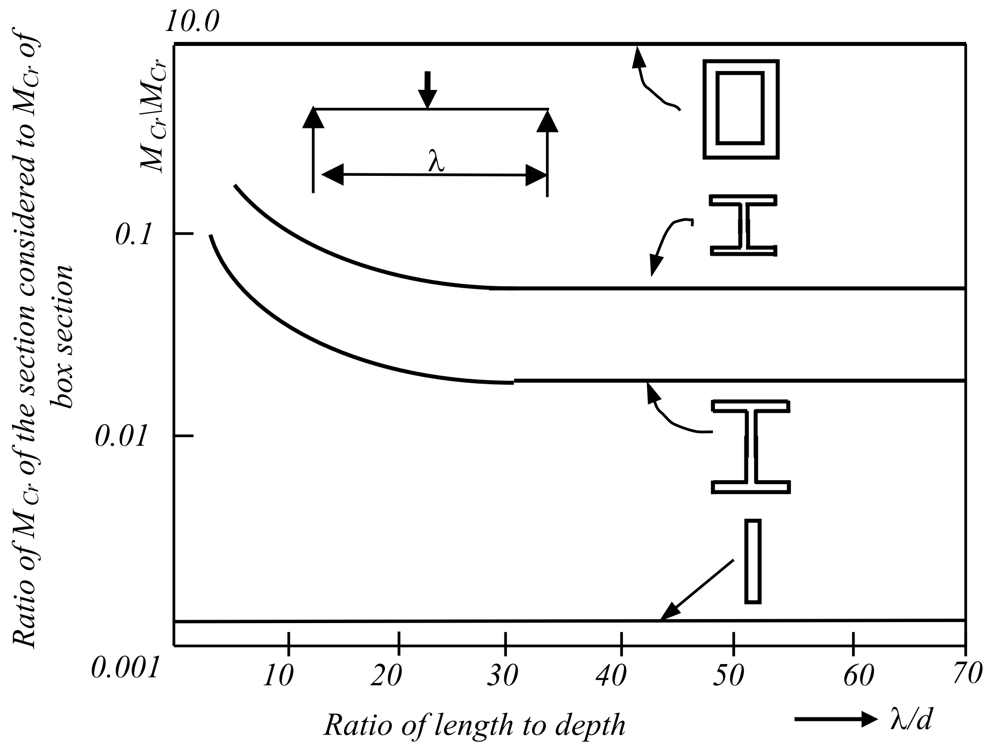


Fig. 12 Effect of type of cross section

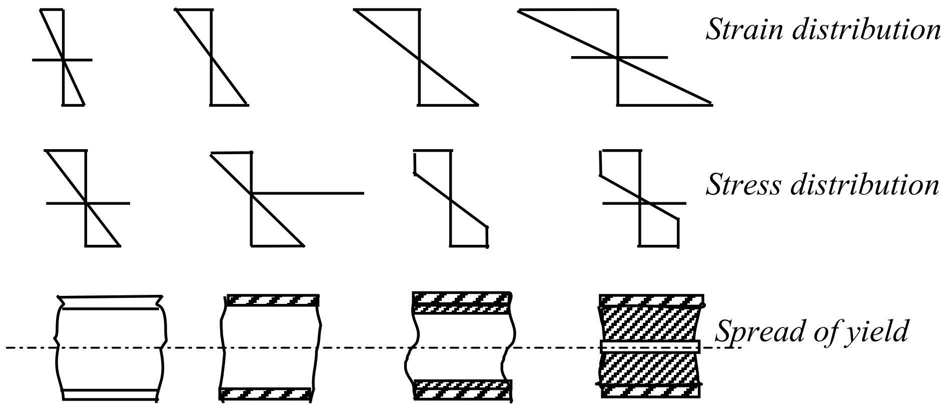
6.0 BUCKLING OF REAL BEAMS

The theoretical assumptions made in section 4.0 are generally not realised in practice. In this section, the behaviour of real beams (which do not meet all the assumptions of the buckling theory) is explained. Effects of plasticity, residual stresses and imperfections are described in the following sections.

6.1 Plasticity effects

Initially, the case, where buckling is not elastic is considered. All other assumptions hold good. As the beam undergoes bending under applied loads, the axial strain distribution at a point in the beam varies along the depth as shown in Fig.13.

With the increase in loading, yielding of the section is initiated at the outer surfaces of the top and bottom flanges. If the M_{cr} of the section as calculated by Eqn.1 is less than M_y, then the beam buckles elastically. In the case where M_{cr} is greater than M_y, some amount of plasticity is experienced at the outer edges before buckling is initiated. If the beam is sufficiently stocky, the beam section attains its full plastic moment capacity, M_p. The interaction between instability and plasticity is shown in Fig. 14.



(Elastic –perfectly plastic material behaviour is assumed)

Fig 13 Strain / Stress Distribution and yielding of section

There are three distinct regions in the curve as given below.

1. Beams with high slenderness ($\sqrt{\frac{M_p}{M_{cr}}} > 1.2$). The failure of the beam is by elastic lateral buckling at M_{cr}
2. Beams of intermediate slenderness $0.4 < \sqrt{\frac{M_p}{M_{cr}}} < 1.2$), where failure occurs by inelastic lateral buckling at loads below M_p and above M_{cr}
3. Stocky beams ($\sqrt{\frac{M_p}{M_{cr}}} < 0.4$)), which attain M_p without buckling.

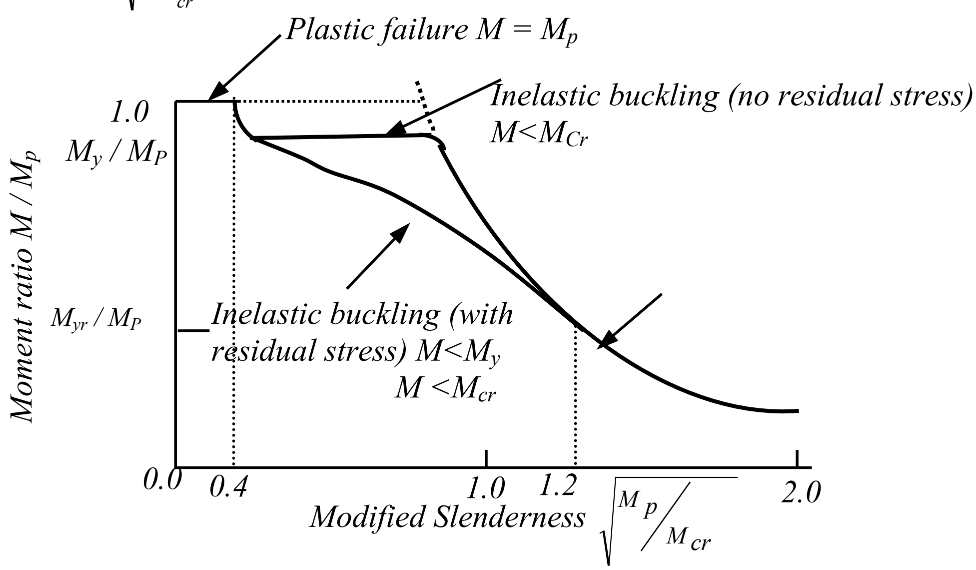


Fig. 14 Interaction between instability and plasticity

6.2 Residual stresses

It is normally assumed that a structural section in the unloaded condition is free from stress and strain. In reality, this is not true. During the process of manufacture of steel sections, they are subjected to large thermal expansions resulting in yield level strains in the sections. As the subsequent cooling is not uniform throughout the section, self-equilibrating patterns of stresses are formed. These stresses are known as *residual stresses*. Similar effects can also occur at the fabrication stage during welding and flame cutting of sections. A typical residual stress distribution in a hot rolled steel beam section is shown in Fig.15.

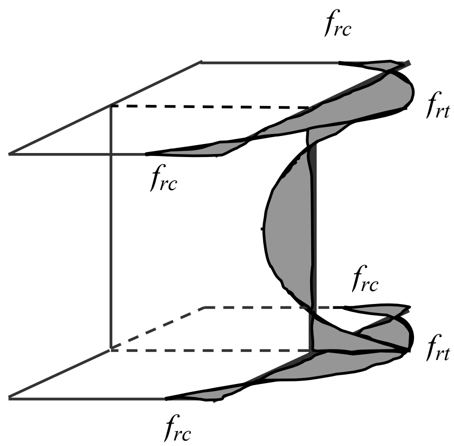


Fig. 15 Residual stresses in I beams

Due to the presence of residual stresses, yielding of the section starts at lower moments. Then, with the increase in moment, yielding spreads through the cross-section. The in-elastic range, which starts at M_{yr} increases instead of the elastic range. The plastic moment value M_p is not influenced by the presence of residual stresses.

6.3 Imperfections

The initial distortion or lack of straightness in beams may be in the form of a lateral bow or twist. In addition, the applied loading may be eccentric inducing more twist to the beam. It is clear that these initial imperfections correspond to the two types of deformations that the beam undergoes during lateral buckling. Assuming $M_{cr} < M_y$, the lateral deflection and twist increase continuously from the initial stage of loading assuming large proportion as M_{cr} is reached. The additional stresses, thus produced, would cause failure of the beam as the maximum stress in the flange tips reaches the yield stress. This form of failure by limiting the stress to yield magnitude is shown in Fig. 16. In the case of beams of intermediate slenderness, a small amount of stress redistribution takes place after yielding and the prediction by the limiting stress approach will be conservative. If residual stresses were also included, the failure load prediction would be conservative even for slender beams.

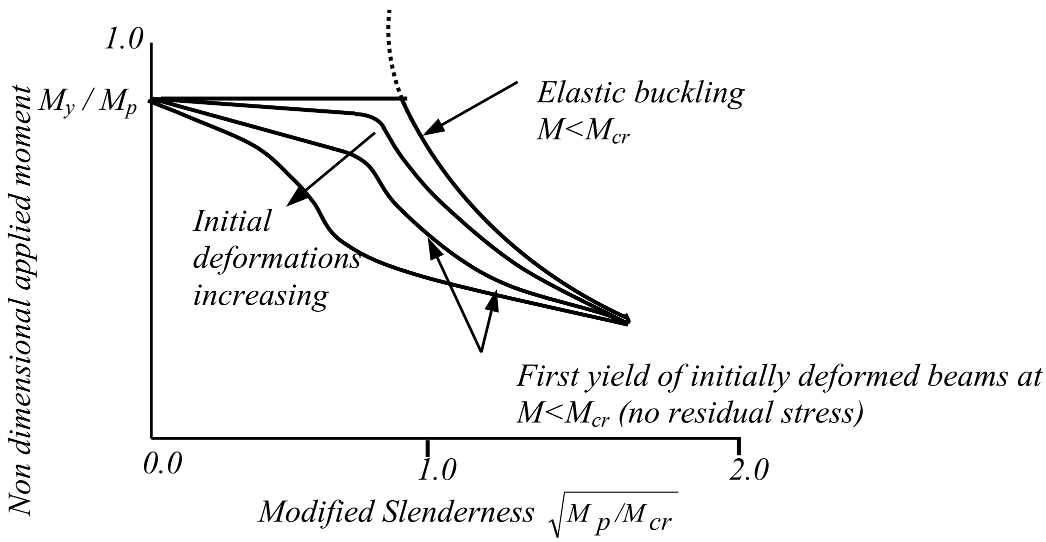


Fig. 16 Beam failure curve

While studying the behaviour of beams, it is necessary to account for the combined effects of the various factors such as instability, plasticity, residual stresses and geometrical imperfections.

7.0 DESIGN APPROACH

Lateral instability is a prime design consideration for all laterally unsupported beams except for the very stocky ones. The value M_{cr} is important in assessing their load carrying capacity. The non-dimensional modified slenderness $\bar{\lambda}_{LT} = \sqrt{M_p / M_{cr}}$ indicates the importance of instability and as a result the governing mode of failure.

For design purposes, the application of the theoretical formula is too complex. Further, there is much difference between the assumptions made in the theory and the real characteristics of the beams. However, as the theoretical prediction is elastic, it provides an upper bound to the true strength of the member. A non-dimensional plot with abscissa as $\sqrt{M_p / M_{cr}}$ and the ordinate as M/M_p, where M_p is the plastic moment capacity of section and M is the failure moment shows clearly the lateral torsional behaviour of the beam. Such a non-dimensional plot of lateral torsional buckling moment and the elastic critical moment is shown in Fig 17. Experiments on beams validate the use of such a curve as being representative of the actual test data.

Three distinct regions of behaviour may be noticed in the figure. They are:

- Stocky, where beams attain M_p, with values of $\bar{\lambda}_{LT} < 0.4$
- Intermediate, the region where beams fail to reach either M_P or M_{cr} ; $0.4 < \bar{\lambda}_{LT} < 1.2$
- Slender, where beams fail at moment M_{cr}; $\bar{\lambda}_{LT} > 1.2$

As pointed out earlier, lateral stability is not a criterion for stocky beams. For beams of the second category, which comprise of the majority of available sections, design is based on inelastic buckling accounting for geometrical imperfections and residual stresses.

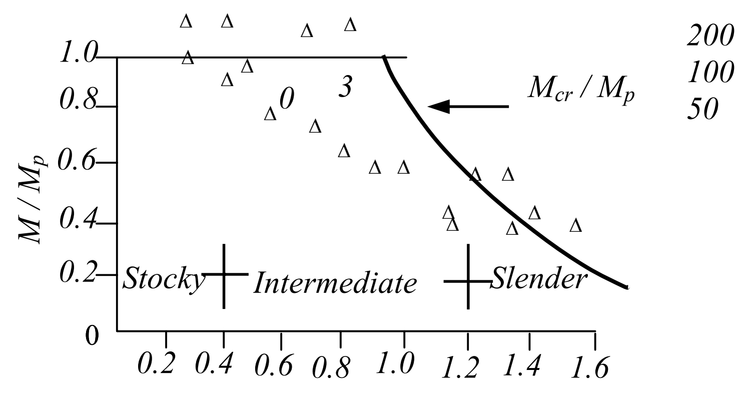


Fig 17. Theoretical elastic critical moment

7.1 Conservative design procedure

The lateral buckling moment capacity of a section can be expressed as

$$M_b = p_b S_x \tag{11}$$

where, p_b is the bending strength accounting for lateral instability
S_x is the appropriate plastic section modulus

The slenderness of the beam λ_{LT} is defined as:

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{p_y}}\ \bar{\lambda}_{LT} \tag{12}$$

This has close similarity to the slenderness associated with compressive buckling of a column. The relation between p_b and λ_{LT} is shown in Fig.18.

In the case of slender beams, p_b is related to λ_{LT} . λ_{LT} can be determined for a given section by the following relationship

$$\lambda_{LT} = n\, u\, v\, \lambda_e / r_y \tag{13}$$

where, n is the slenderness correction factor

u is buckling parameter from steel tables (= 0.9 for rolled beams and channels and1.0 for other sections)

v is slenderness factor and $f(\lambda/r_y, x)$, given in Table 14 of BS 5950 part 1; but pproximated to 1.0 for preliminary calculations

x is the torsional index which is provided in BS 5950 part 1

$$x = 0.566\, h\,(A/\ J)^{1/2}$$ for bi-symmetric sections and sections symmetric about minor axis, and

$$x = 1.132\,\left(A\,H/\ I_y J\right)^{1/2}$$ for sections symmetric about major axis.

where

A is the cross sectional area of the member.
I_y is the second moment of the area about the minor axis
H is the warping constant
J is the torsion constant
h is the distance between the shear center of the flanges.

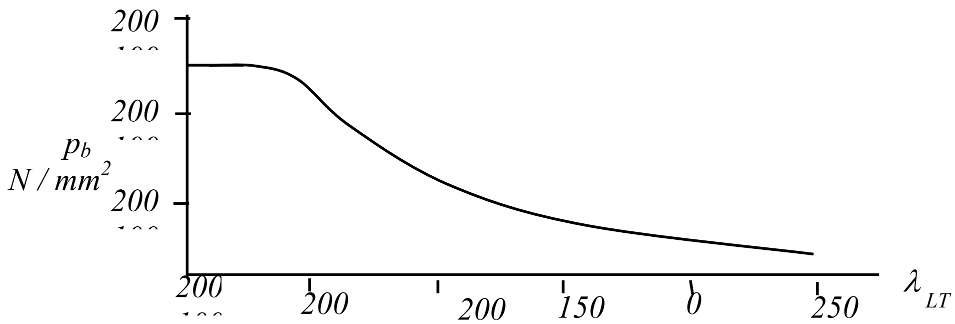


Fig. 18 Bending strength for rolled sections of design strength 240 N / mm²

For compact sections, full plasticity is developed at the most heavily stressed section. Unlike plastic design, moment redistribution is not considered here. For example, for a particular grade of steel and for $\bar{\lambda}_{LT} < 0.4$, when p_b attains the value of p_y, $\lambda_{LT} = 37$. Hence, this is the value of maximum slenderness for which instability does not influence strength.

A good design can be achieved by determining the value of λ_{LT} and thereby p_b more accurately. M_b can be determined using Eqn.11. Effective lengths of the beam may be adopted as per the guidelines given in Table 2. For beams, and segments of beams between lateral supports, equivalent uniform moments may be calculated to determine their relative severity of instability. The lateral stability is checked for an equivalent moment $\bar{M}$ given by

$$\bar{M} = m\, M_{max} \qquad (14)$$

where m is the equivalent uniform moment factor.

If $M_b > \bar{M}$, the section chosen is satisfactory. At the heavily stressed locations, local strength should be checked against development of M_p.

$$M_{max} \ngtr M_p \qquad (15)$$

7.2 Design approach as per New IS: 800:

The New IS: 800 follows the same design philosophy with certain alterations in the parameters for calculating design bending strength governed by lateral torsional buckling. The step by step design procedure has been detailed below:

The design bending strength of laterally unsupported beam as governed by lateral torsional buckling is given by:

$M_d = \beta_b Z_p f_{bd}$

[β_b = 1.0 for plastic and compact sections

= Z_e/Z_p for semi-compact sections

Z_p, Z_e = plastic section modulus and elastic section modulus with respect to extreme compression fibre.]

f_{bd} = design bending compressive stress, obtained as given below:

f_{bd} = $\chi_{LT} f_y / \gamma_{m0}$

χ_{LT} = bending stress reduction factor to account for lateral torisonal buckling

$$\chi_{LT} = \frac{1}{\left\{\phi_{LT} + \left[\phi_{LT}^2 - \lambda_{LT}^2\right]^{0.5}\right\}} \le 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\lambda_{LT} - 0.2\right) + \lambda_{LT}^{\ 2}\right]$$

α_{LT}, the imperfection parameter is given by:

α_{LT} = 0.21 for rolled steel section

α_{LT} = 0.49 for welded steel section

The non-dimensional slenderness ratio, λ_{LT}, is given by

$$\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}}$$

$$= \sqrt{f_y / f_{cr,b}}$$

M_{cr} = elastic critical moment to be calculated as per **8.2.2.1**

$f_{cr,b}$ = extreme fibre bending compressive stress corresponding to elastic lateral buckling moment (**8.2.2.1**, Table 8.1)

8.0 SUMMARY

Unrestrained beams that are loaded in their stiffer planes may undergo lateral torsional buckling. The prime factors that influence the buckling strength of beams are: the un braced span, cross sectional shape, type of end restraint and the distribution of moment. For the purpose of design, the simplified approach as given in BS: 5950 Part-1 has been presented. The effects of various parameters that affect buckling strength have been accounted for in the design by appropriate correction factors. The behaviour of real beams (which do not comply with the theoretical assumptions) has also been described. In order to increase the lateral strength of a beam, bracing of suitable stiffness and strength has to be provided.

9.0 REFERENCES

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3. Martin L.H. and Purkiss J.A., ‘Structural design of steel work to BS 5950, Edward Arnold, 1992.

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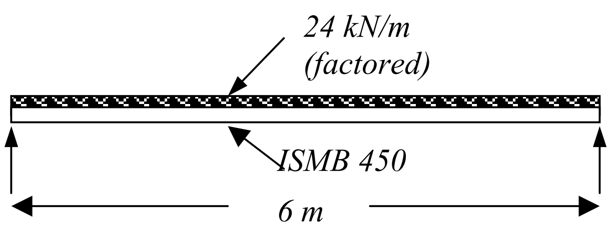
		Checked by. *TKB*	*Date. 26/02/07*

Problem - 1

Check the adequacy of ISMB 450 to carry a uniformly distributed load of 24 kN / m over a span of 6 m. Both ends of the beam are attached to the flanges of columns by double web cleat.

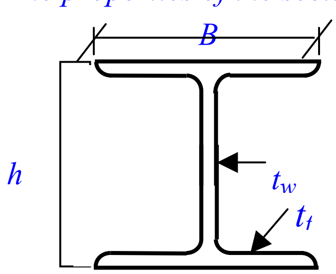


Design check:

For the end conditions given, it is assumed that the beam is simply supported in a vertical plane, and at the ends the beam is fully restrained against lateral deflection and twist with, no rotational restraint in plan at its ends.

Section classification of ISMB 450:

The properties of the section are:



Depth, h = 450 mm

Width, B = 150 mm

Web thickness, t_w = 9.4 mm

Flange thickness, t_f = 17.4 mm

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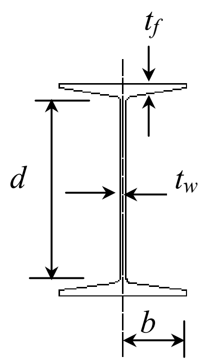

Rolled Steel Beams

Depth between fillets, d = 379.2 mm

Radius of gyration about minor axis, r_y = 30.1 mm

Plastic modulus about major axis, Z_p = 1533.36 x 10^3 mm^3 — *Appendix I of IS: 800*

Assume f_y = 250 N/mm², E=200000 N/mm², γ_m = 1.10,

(I) Type of section

Flange criterion:

$$b = \frac{B}{2} = \frac{150}{2} = 75\ mm$$

$$\frac{b}{t_f} = \frac{75.0}{17.4} = 4.31$$

$$\frac{b}{t_f} < 9.4\varepsilon \quad where\ \varepsilon = \sqrt{\frac{250}{f_y}}$$

Hence O.K.

Web criterion:

$$\frac{d}{t_w} = \frac{379.2}{9.4} = 40.3$$

$$\frac{d}{t_w} < 84\,\varepsilon$$

Hence O.K.

Since $\frac{b}{t_f} < 9.4\varepsilon$ *and* $\frac{d}{t_w} < 84\varepsilon$, *the section is classified as 'plastic'* — *Table 3.1 (Section 3.7.2) of IS: 800*

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(II) Check for lateral torsional buckling:

Check for Slenderness Ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against Warping, effective length of simply supported beam $L_{LT} = 1.0\ L$, where L is the span of the beam.

Table 8.3 of IS: 800

Hence, $L_{LT} = 1.0 \times 6.0\ M = 6000\ mm$, $L_{LT}/r = 6000/30.1 = 199.33$

And $h/t_f = 450/17.4 = 25.86$,

Corresponding value of Critical Stress, $f_{cr,b} = 99.47\ N/mm^2$

Table 8.2 of IS: 800

For $f_{cr,b} = 99.47\ N/mm^2$, $f_{bd} = 76.94\ N/mm^2$ *for* $\alpha_{LT} = 0.21$ *for rolled steel section*

Table 8.1a of IS: 800

Now, $M_d = \beta_b Z_p f_{bd}$

where

β_b = 1.0 for plastic and compact sections
= Z_e/Z_p for semi-compact sections

Z_p, Z_e = plastic section modulus and elastic section modulus with respect to extreme compression fibre.

f_{bd} = design bending compressive stress, obtained as given in Table 8.1a of New IS: 800

Hence, $M_d = \beta_b Z_p f_{bd} = 1.0 \times 1533.36 \times 76.94/1000 = 117976.72/1000$

$= 117.98\ kN\text{-}m$

Hence, Bending strength, $M_d = 117.98\ kN\text{-}m$

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For the simply supported beam of 6.0 m span with a factored load of 24.0 KN/m

$$M_{max} = \frac{w\lambda^2}{8} = \frac{24*6^2}{8}$$

$$= 108.0 \text{ KN m} < 117.98 \text{ kN m}$$

$$\text{Hence } M_d > M_{max}$$

$\therefore$ *ISMB 450 is adequate against lateral torsional buckling.*

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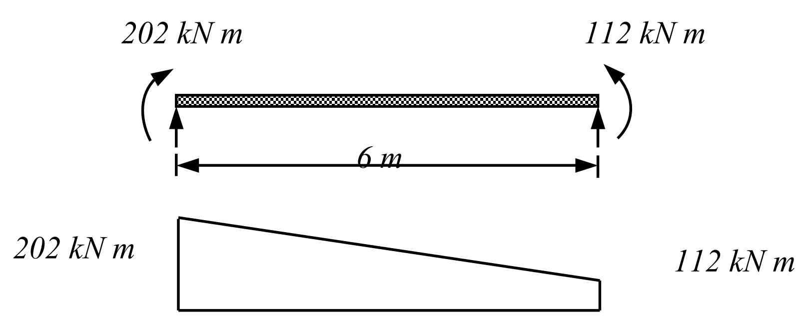
Problem-2

(i) A simply supported beam of span 6 m is subjected to end moments of 202 kN m (clockwise) and 112 k N m (anticlockwise) under factored -applied loading. Check whether ISMB 450 is safe with regard to lateral buckling.



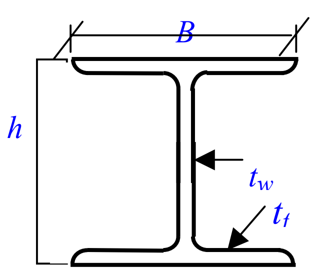
B.M Diagram

Design check:

For the end conditions given, it is assumed that the beam is simply supported in a vertical plane, and at the ends the beam is fully restrained against lateral deflection and twist with, no rotational restraint in plan at its ends.

Section classification of ISMB 450

The properties of the section are:



Depth, $h = 450$ mm

Width, $B = 150$ mm

Web thickness, $t_w = 9.4$ mm

Flange thickness, $t_f = 17.4$ mm

$I_y = 834 \times 10^4$ mm^4

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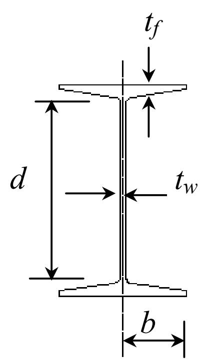

Rolled Steel Beams

Depth between fillets, $d = 379.2$ mm

Radius of gyration about minor axis, $r_y = 30.1$ mm

Plastic modulus about major axis, $Z_p = 1533.36 \times 10^3$ mm^3 — Appendix I of IS: 800

Assume $f_y = 250$ N/mm^2, $E=200000$ N/mm^2, $\gamma_m = 1.10$,

(II) Type of section

Flange criterion:

$$b = \frac{B}{2} = \frac{150}{2} = 75\ mm$$

$$\frac{b}{t_f} = \frac{75.0}{17.4} = 4.31$$

$$\frac{b}{t_f} < 9.4\varepsilon \quad where\ \varepsilon = \sqrt{\frac{250}{f_y}}$$

Hence O.K.

Web criterion:

$$\frac{d}{t_w} = \frac{379.2}{9.4} = 40.3$$

$$\frac{d}{t_w} < 84\varepsilon$$

Hence O.K.

Since $\frac{b}{t_f} < 9.4\varepsilon$ and $\frac{d}{t_w} < 84\varepsilon$, the section is classified as 'plastic' — Table 3.1 (Section 3.7.2) of IS: 800

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(II)Check for lateral torsional buckling:

Check for Slenderness Ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against Warping, effective length of simply supported beam $L_{LT} = 1.0\ L$, where L is the span of the beam. — *Table 8.3 of IS: 800*

Hence, $L_{LT} = 1.0 \times 6.0\ M = 6000\ mm$, $L_{LT}/r = 6000/30.1 = 199.33$

Since the moment is varying from 155 k-Nm to 86 k-Nm, there will be moment gradient. So for calculation of f_{bd}*, Critical Moment,* M_{cr} *is to be calculated.*

Now, Critical Moment, — *Clause F.1.2 of Appendix F of IS: 800*

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2}\left\{\left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + \left(c_2\, y_g - c_3\, y_j\right)^2\right]^{0.5} - \left(c_2\, y_g - c_3\, y_j\right)\right\}$$

Where,

c_1, c_2, c_3 = factors depending upon the loading and end restraint conditions (Table F.1)

K, K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports,

Here, both K *and* K_w can be taken as 1.0.

and

y_g = *y distance between the point of application of the load and the shear centre of the cross section and is positive when the load is acting towards the shear centre from the point of application*

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$y_j = y_s - 0.5 \int_A (z^2 - y^2)\, y\, dA / I_z$

y_s = *coordinate of the shear centre with respect to centroid, positive when the shear centre is on the compression side of the centroid*

Here, for plane and equal flange I section,

$y_g = 0.5 \times h = 0.5 \times 0.45 = 0.225\ M = 225\ mm$

$y_j = 1.0\,(2\beta_f - 1)\, h_y / 2.0$ (when $\beta_f \leq 0.5$)

h_y = distance between shear centre of the two flanges of the cross section = $h - t_f$

here, $\beta_f = 0.5$, *and* $h_y = h - t_f = 450 - 17.4 = 432.6\ mm$

hence, $y_j = 1.0 \times (2 \times 0.5 - 1) \times 432.6/2.0 = 0$

and $y_s = 0$

$I_t = \sum b_i t_i^3 / 3$, for open section

$= 2 \times 150 \times 17.4^3 + (450 - 2 \times 17.4) \times 9.4^3 = 192.527 \times 10^4\ mm^4$

The warping constant, I_w, is given by

$I_w = (1-\beta_f)\, \beta_f\, I_y\, h_y^2$ for I sections mono-symmetric about weak axis

$= (1-0.5) \times 0.5 \times 834 \times 10^4 \times 432.6^2 = 39019265.46 \times 10^4\ mm^6$

Modulus of Rigidity, $G = 0.769 \times 10^5 N/mm^2$

Here, $\psi = 86/155 = 0.555$ *and* $K = 1.0$, *for which* — *Table F.1 of Appendix F of IS: 800*

$c_1 = 1.283$, $c_2 = 0$ *and* $c_3 = 0.993$

Hence, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 E I_y}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \frac{G I_t (KL)^2}{\pi^2 E I_y} + \left(c_2\, y_g - c_3\, y_j \right)^2 \right]^{0.5} - \left(c_2\, y_g - c_3\, y_j \right) \right\}$$

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		Checked by. *TKB*	Date.*28/02/07*

$$= 1.283\frac{\pi^2 x200000x834x10^4}{(1.0x6000)^2}\left\{\left[\left(\frac{1}{1}\right)^2\frac{39019265x10^4}{834x10^4}+\frac{0.769x10^5 x192.527x10^4 x6000^2}{\pi^2 x200000x834x10^4}\right]^{0.5}\right\}$$

= 357142.72 x 10^3 N-mm

Calculation of f_{bd}:

Now, $\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}} = \sqrt{1.0x1533.36x10^3 x250/357142.72x10^3}$ *= 1.036* *(Clause 8.2.2 of IS: 800)*

for which, $\phi_{LT} = 0.5x\left[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2\right]$

$= 0.5x\left[1+0.21(1.036-0.2)+1.036^2\right] = 1.124$

for which, $\chi_{LT} = \frac{1}{\left\{\phi_{LT}+\left[\phi^2{}_{LT}-\lambda^2{}_{LT}\right]^{0.5}\right\}} = \frac{1}{\left\{1.124+\left[1.124^2-1.036^2\right]^{0.5}\right\}}$

$= 0.641$

$f_{bd} = \chi_{LT} f_y / \gamma_{m0}$ *= 0.641 x 250 / 1.10 = 145.68 N/mm^2*

Hence, $M_d = \beta_b Z_p f_{bd}$ *=1.0 x 1533.36 x 145.68/1000 = 223379.88/1000*

= 223.38 kN-m

Max. Bending Moment, M_{max} *= 202 kN-m*

Hence, $M_d > M_{max}$ *(223.38 > 202)*

$\therefore$ *ISMB 450 is adequate against lateral torsional buckling for the applied bending moments.*

(ii) If the beam of problem (i) is subjected to a central load producing a maximum factored moment of 202 kN m, check whether the beam is still safe.

For this problem with zero bending moments at the supports and central max bending moment being 202 kN-m, *(Table F.1 of Appendix F of IS: 800)*

For the value of K = 1.0, $c_1 = 1.365$; $c_2 = 0.553$ *and* $c_3 = 1.780$

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		Checked by. *TKB*	Date.*28/02/07*

Hence, Critical Moment,

$$M_{cr} = c_1 \frac{\pi^2 EI_y}{(KL)^2}\left\{\left[\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{GI_t (KL)^2}{\pi^2 EI_y} + \left(c_2\ y_g - c_3\ y_j\right)^2\right]^{0.5} - \left(c_2\ y_g - c_3\ y_j\right)\right\}$$

$$= 1.365\frac{\pi^2 x2x10^5 x834x10^4}{(1.0x6000)^2}\left\{\left[\frac{390.19x10^9}{834x10^4} + \frac{0.769x10^5 x192527x10^4 x36x10^6}{\pi^2 x2x10^5 x834x10^4} + (0.553x225)^2\right]^{0.5} - 0.553x225\right\}$$

=310158.31 x 10^3N-mm

Calculation of f_{bd}:

Now, $\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}} = \sqrt{1.0x1533.36x10^3 x250/310158.31x10^3}$

$= 1.112$

for which, $\phi_{LT} = 0.5x\left[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}{}^2\right]$

$= 0.5x\left[1 + 0.21(1.112 - 0.2) + 1.112^2\right] = 1.214$

for which, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT} + \left[\phi^2{}_{LT} - \lambda^2{}_{LT}\right]^{0.5}\right\}} = \dfrac{1}{\left\{1.214 + \left[1.214^2 - 1.112^2\right]^{0.5}\right\}}$

$= 0.588$

$f_{bd} = \chi_{LT} f_y / \gamma_{m0} = 0.588 \times 250 / 1.10 = 133.64\ N/mm^2$

Hence, $M_d = \beta_b Z_p f_{bd} = 1.0 \times 1533.36 \times 133.64/1000 = 204918.23/1000$

$= 204.92\ kN\text{-}m$

Therefore the M_d > M_{max} (204.92 > 202)

Therefore the section ISMB 450 is adequate against lateral torsional buckling for this case also.

Clause 8.2.2 of IS: 800